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The Role of Numerical Scheme Accuracy in WMLES of the Aerodynamics of the Windsor Body

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Introduction

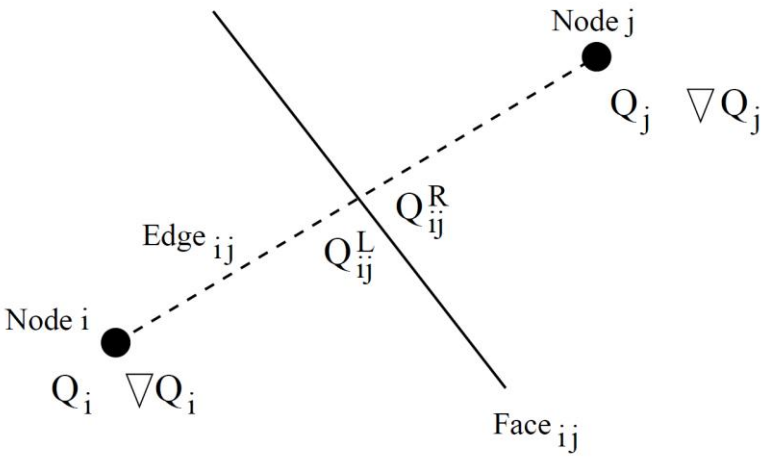
WMLES enables simulations of industrial flows at affordable computational cost, but their accuracy remains sensitive to numerical dissipation and dispersion.

Therefore, in this work:

- A sixth-order extension of the original UMUSCL formulation is proposed for incompressible solvers.
- A curvature-aware TVD limiter is introduced for robust application on unstructured meshes.
- The formal accuracy is verified using smooth two-dimensional scalar transport.
- The impact on WMLES is assessed for turbulent channel flow and the Windsor body.

Original UMUSCL scheme

$$w_f^L = w_j + \frac{\kappa}{2}\left(w_k - w_j\right) + (1-\kappa)\left(\mathbf{x}_f - \mathbf{x}_j\right)\cdot\nabla w_j$$



- The face states combine cell-to-cell interpolation with gradient extrapolation.
- The left and right reconstructions are symmetric.
- Setting ($\kappa = 1/3$) gives the fourth-order central UMUSCL operator on a uniform grid.
- The compact reconstruction is directly applicable to unstructured finite-volume meshes.

Reference: Burg, C. O. E., “Higher Order Variable Extrapolation for Unstructured Finite Volume RANS Flow Solvers,” AIAA Paper 2005-4999, 2005.

UMUSCL6 through successive differentiation

This formulation builds on the successive-differentiation approach of Yang and Harris (2015), who obtained a one-sided fourth-order reconstruction. The present work extends this procedure to obtain a sixth-order central operator.

$$w_f^L = w_j + \frac{1-\kappa_u-\kappa_d}{2}\left(w_k - w_j\right) + \kappa_u \hat{\partial}_j w_j + \kappa_d \hat{\partial}_j w_k + \kappa_c \hat{\partial}_j^2 w_j$$

$$\hat{\partial}_j = \left(\mathbf{x}_f - \mathbf{x}_j\right) \cdot \nabla$$

- The first and second directional derivatives are obtained by successive application of the standard finite-volume gradient operator.

Considerations on the effective order of accuracy in the complete solver

- Sixth-order accuracy applies to the central convective operator for a linear equation on a uniform grid.
- For nonlinear Navier–Stokes fluxes, high-order solution reconstruction alone generally remains within the second-order family.
- Second-order time integration and conventional pressure–velocity coupling further limit the accuracy of a solver.
- UMUSCL6 is therefore intended to reduce the leading discretization error and improve spectral resolution within an overall second-order framework.

Reference: Nishikawa, H., “Economically high-order unstructured-grid methods: Clarification and efficient FSR schemes,” International Journal for Numerical Methods in Fluids, 2021.

Extended TVD Limiting

The UMUSCL6 face value is normalized:

$$\psi = 2\frac{w_{f,\mathrm{UMUSCL6}} - w_j}{w_k - w_j}$$

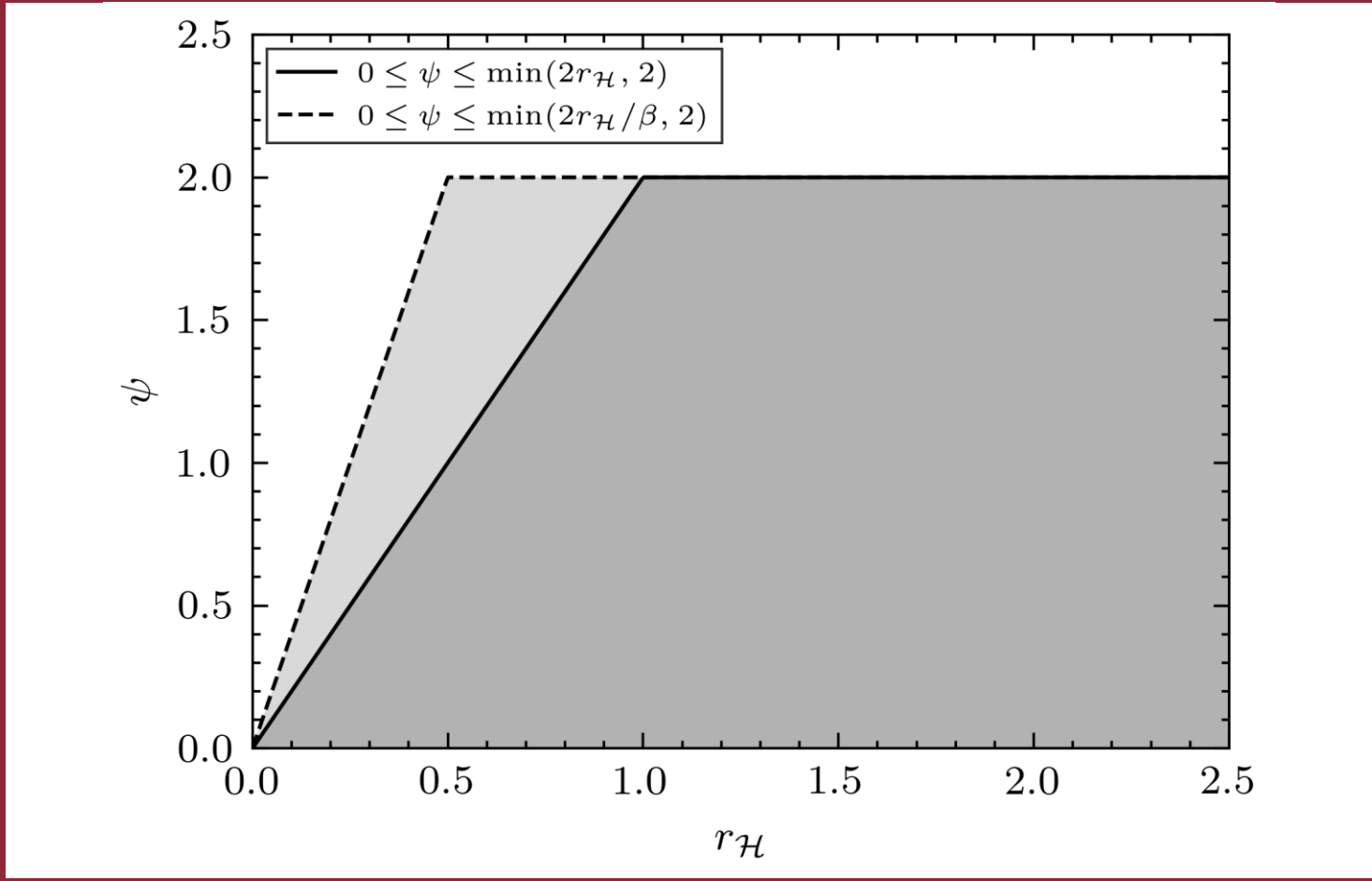
The standard gradient ratio is extended using the directional curvature:

$$r_{\mathcal{H}} = 2\frac{\mathbf{d}\cdot\nabla w_j + \frac{1}{2}\mathcal{H}_{jk}}{w_k - w_j} - 1$$

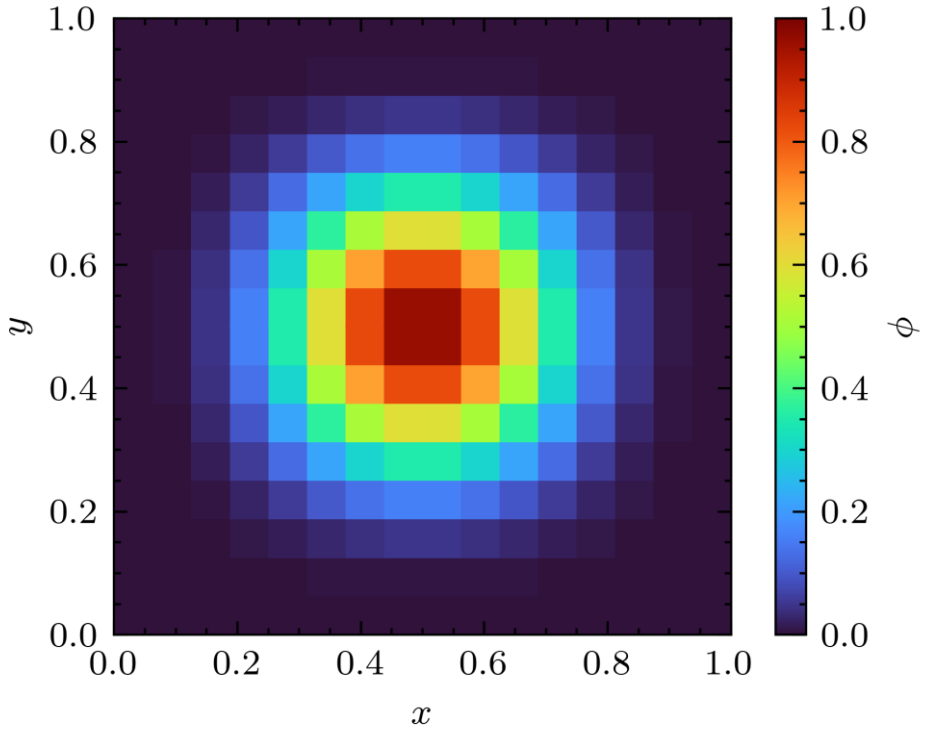
Finally, the reconstructed face value is:

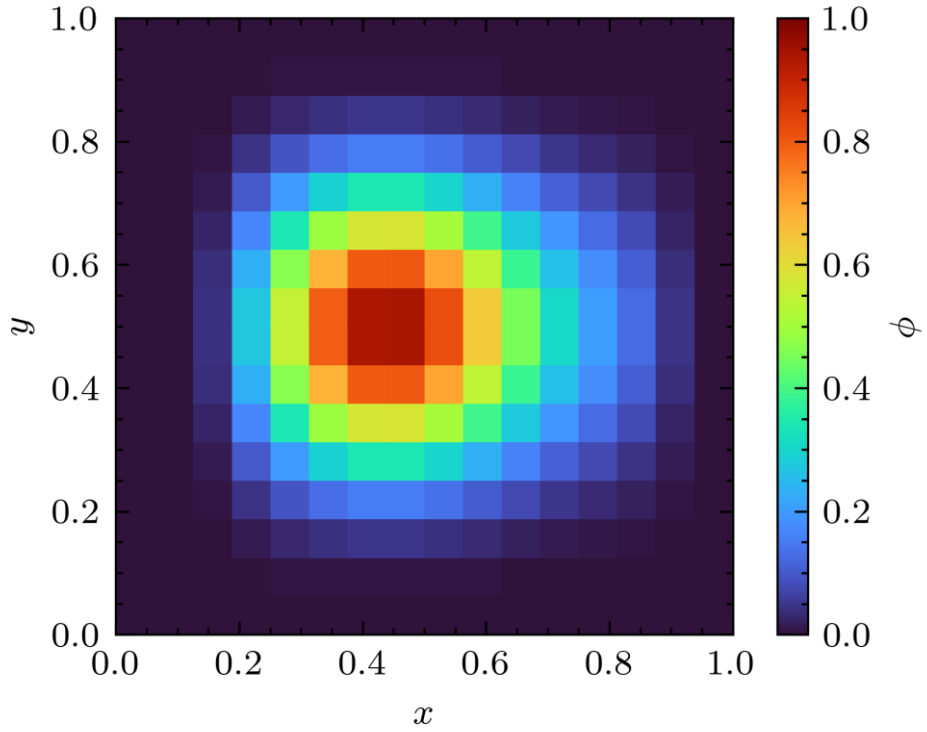
$$w_f = w_j + \frac{1}{2}\psi_{\mathrm{clipped}}(r_{\mathcal{H}})(w_k - w_j)$$

The resulting scheme is referred to as **limitedUMUSCL6**.



2D smooth scalar transport – UMUSCL6 vs second order central

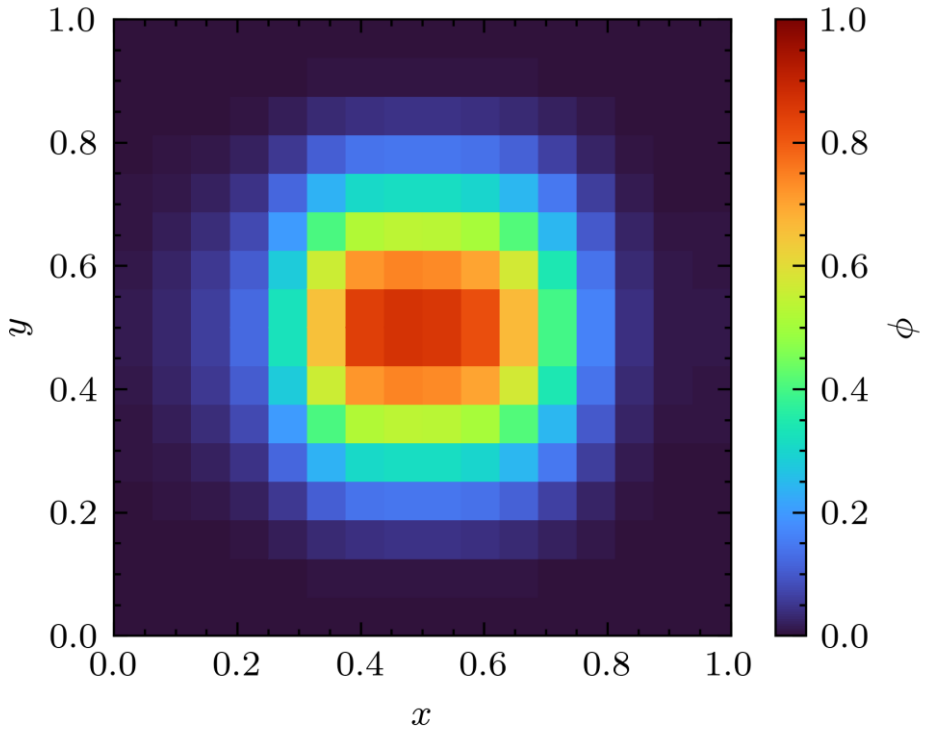


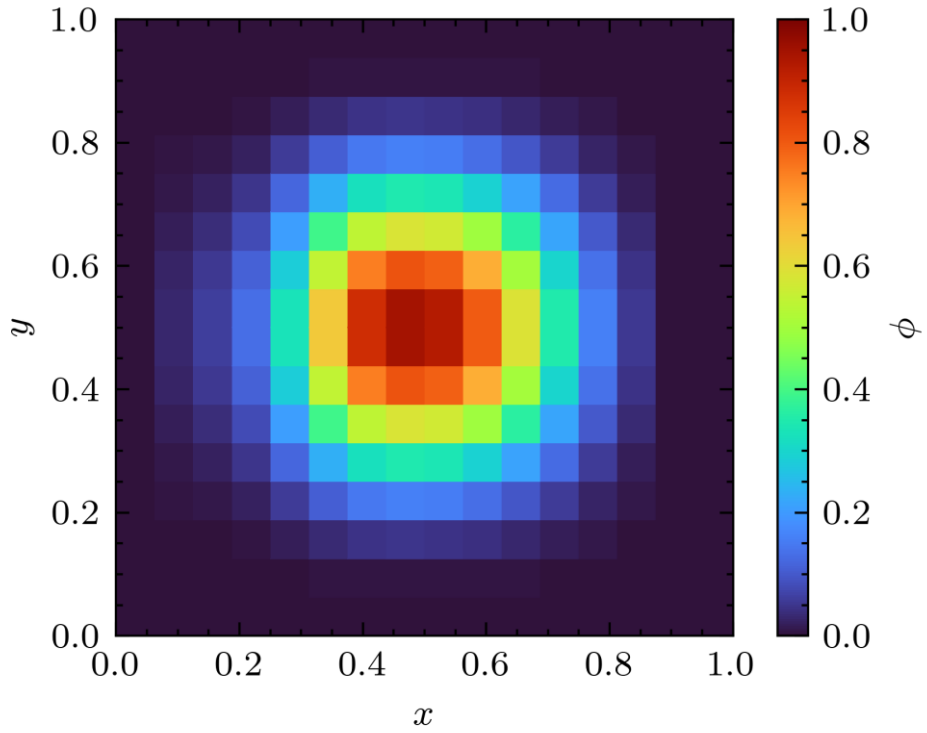


Smooth scalar profile transported once through the periodic unit-square domain.

At the same mesh resolution, UMUSCL6 provides a more accurate result.

2D smooth scalar transport – limitedUMUSCL6 r vs $r_{\mathcal{H}}$

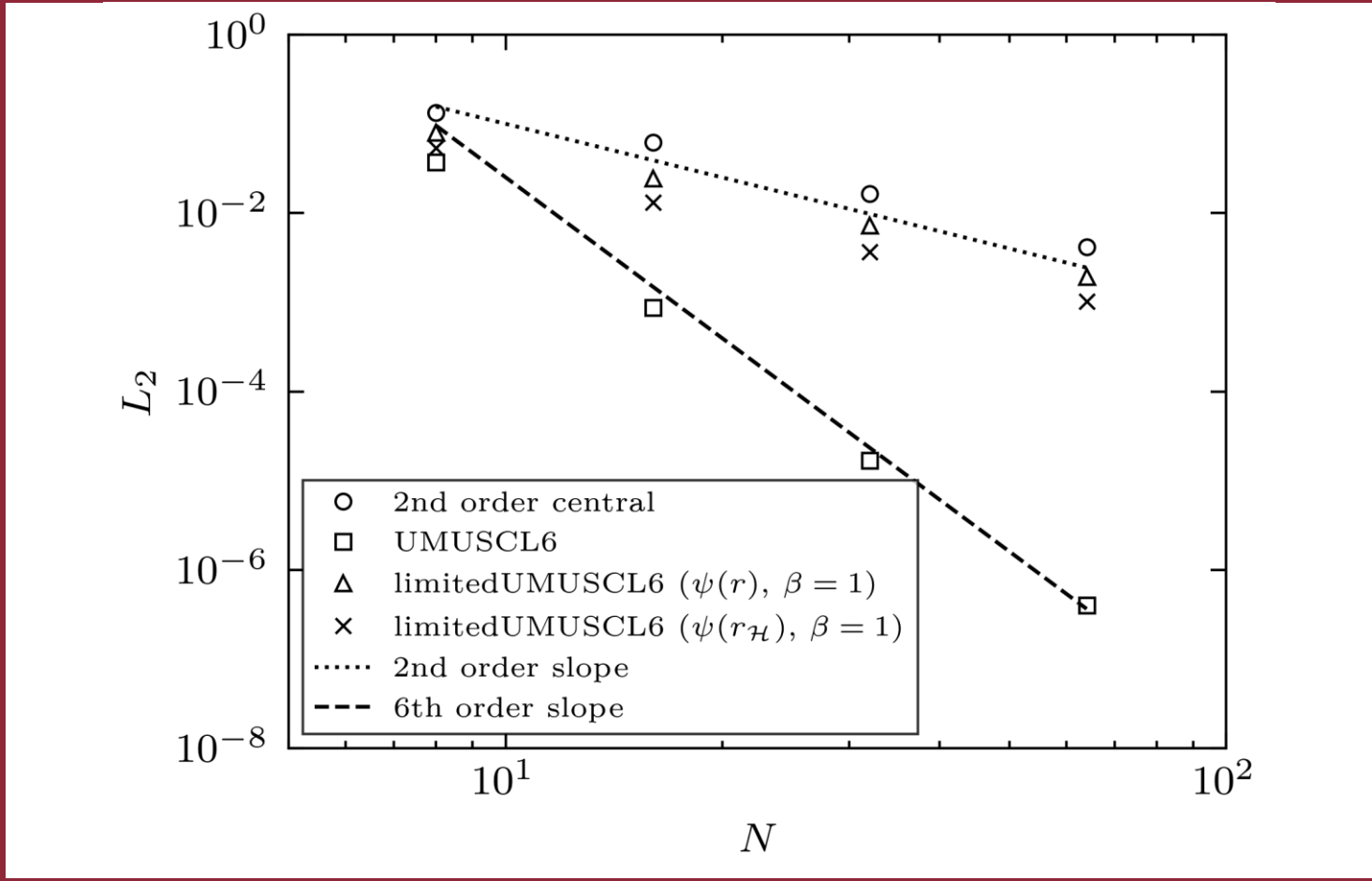




Both formulations remain second-order when limiting is active, but the curvature-aware ratio substantially reduces the error magnitude.

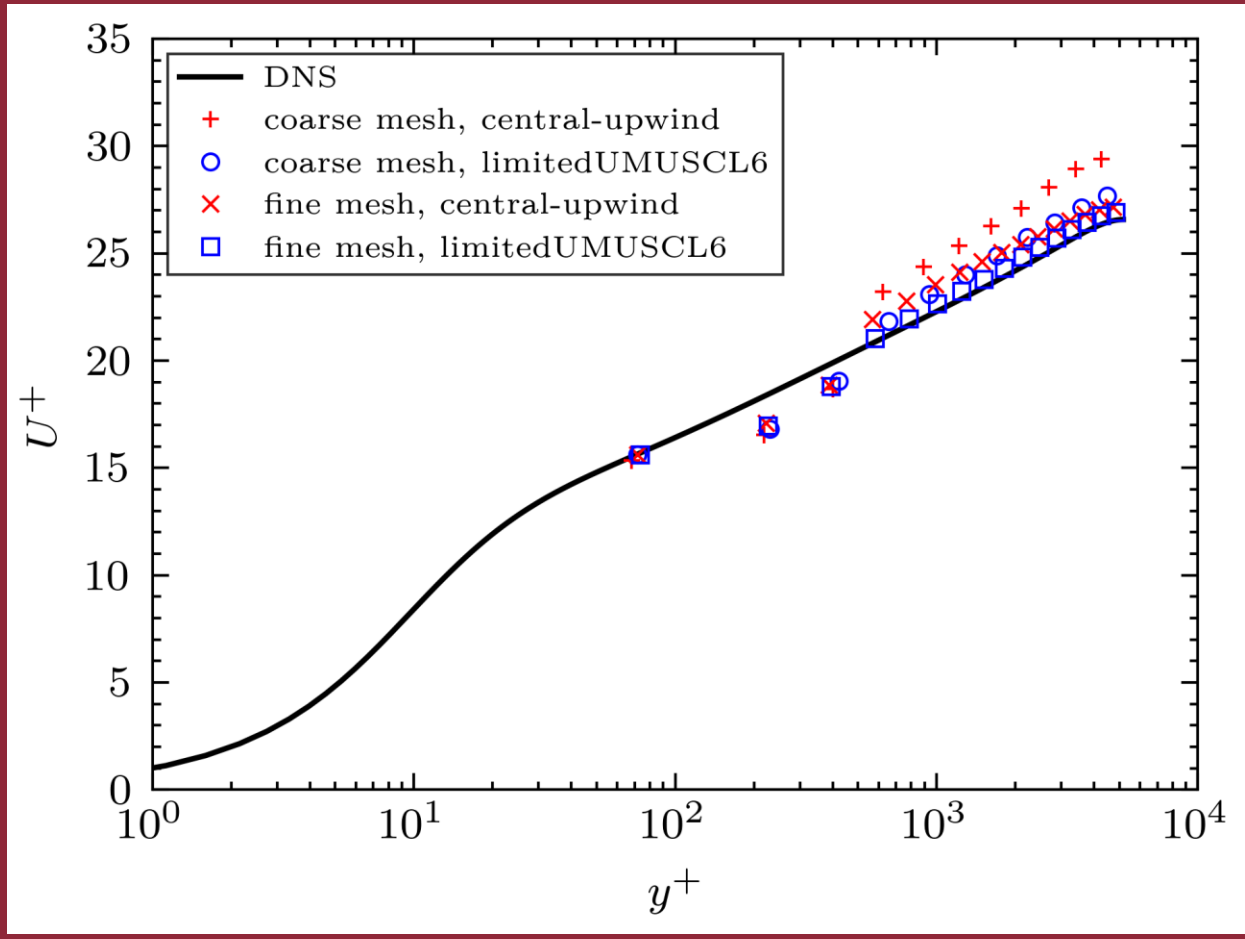
2D smooth scalar transport – Grid convergence

- L_2 error, $N = 8,16,32,64$
- UMUSCL6 follows the sixth-order reference slope, confirming the theoretical analysis.
- The second-order central and limitedUMUSCL6 schemes follow the second-order slope.
- Curvature-aware limiting with gives a consistently smaller error.



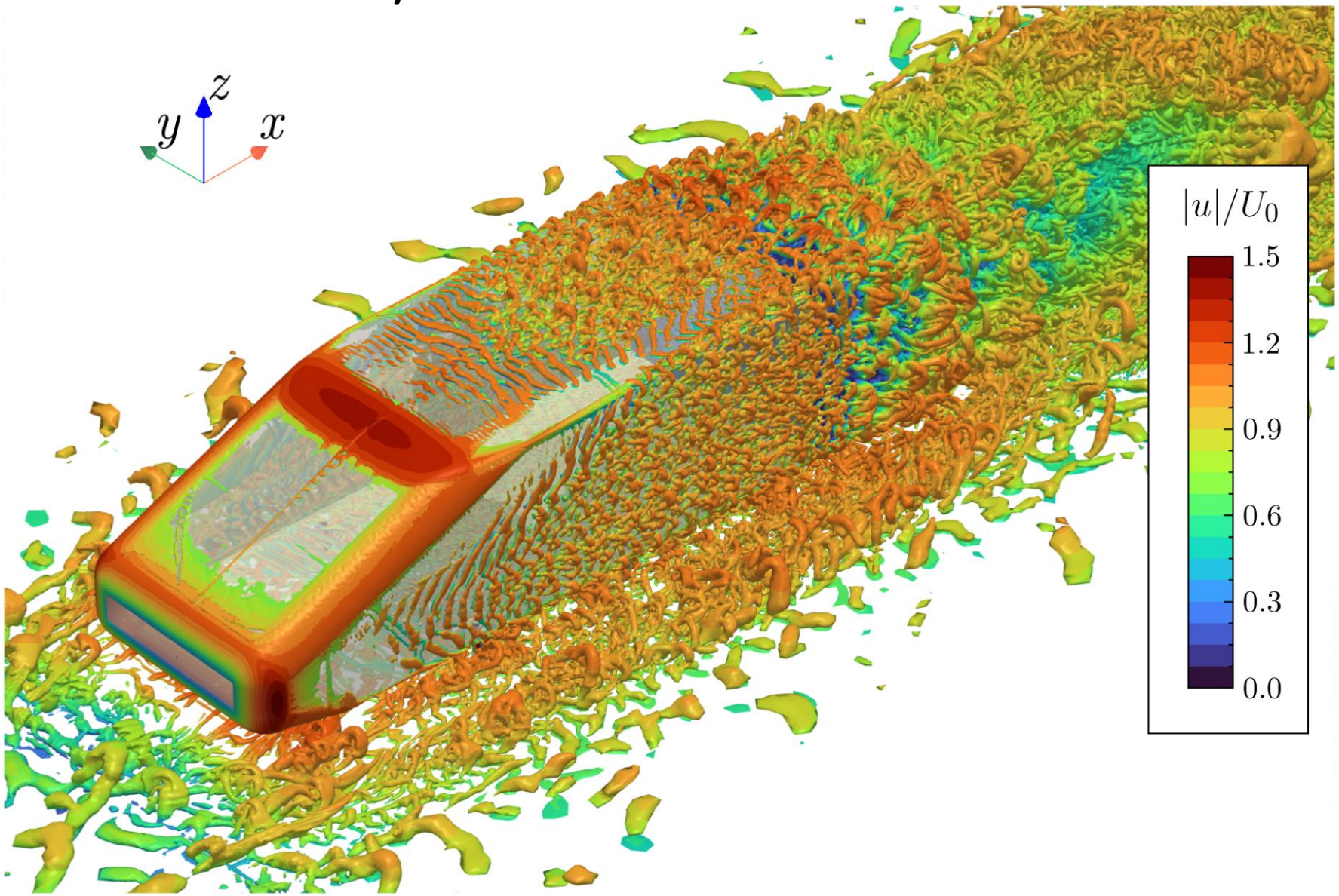
WMLES of turbulent channel flow

- Fully developed channel flow at $Re_\tau = 5200$.
- Two grids are considered: $32 \times 22 \times 16$ and $56 \times 32 \times 28$, both with $y_1^+ \approx 70$.
- The WMLES combines the σ SGS model, a Spalding-law wall-stress condition and a near-wall mixing-length viscosity correction.
- limitedUMUSCL6 with $r_{\mathcal{H}}$ and $\beta = 0.2$ is compared with a 50% linear and 50% linearUpwind blend.
- On the coarse grid, limitedUMUSCL6 reduces the log-layer mismatch and improves the outer-region velocity profile.
- The difference decreases with grid refinement.



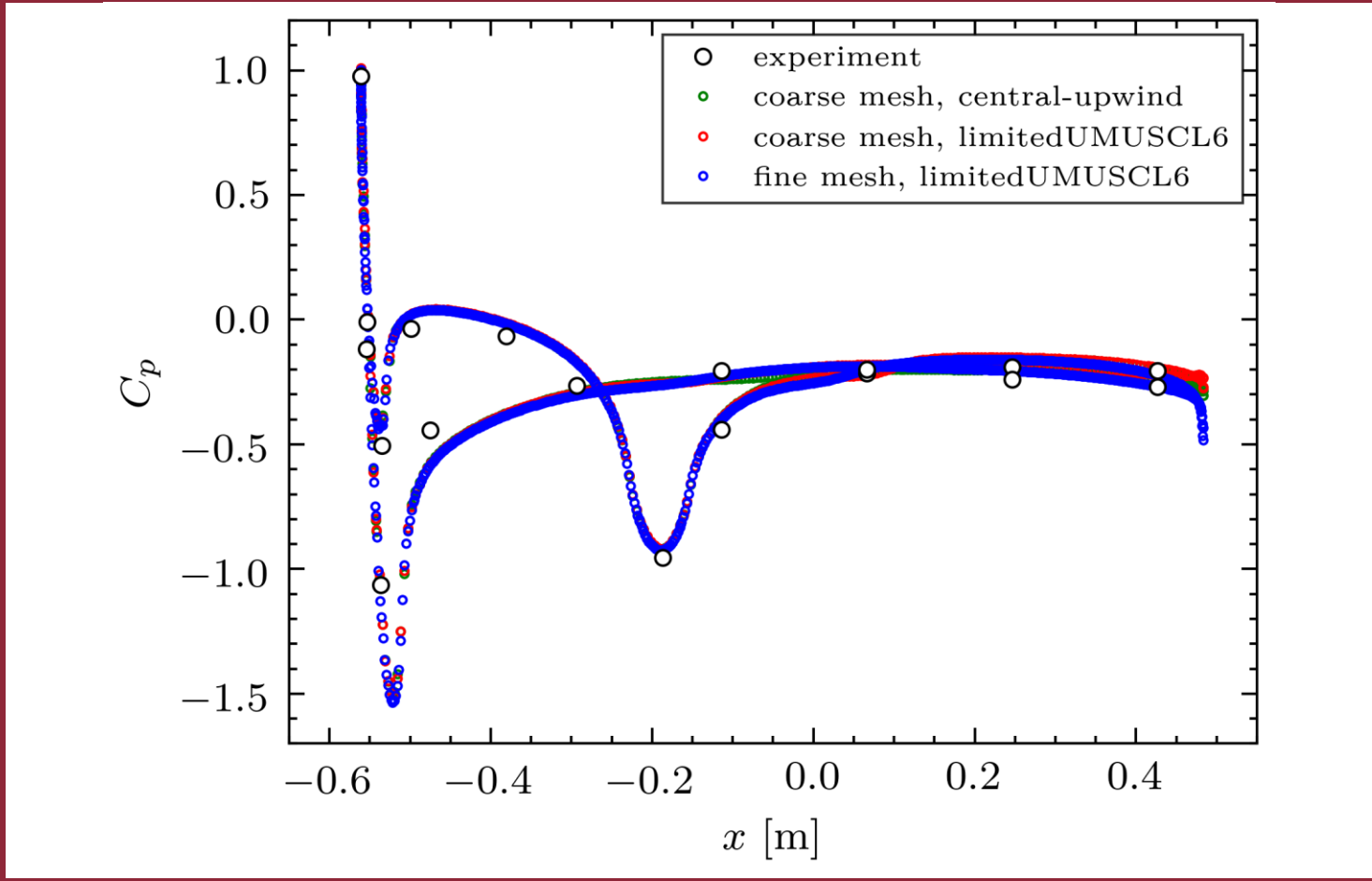
References: Lee and Moser (2015); Angelino et al. (2020).

Windsor Body

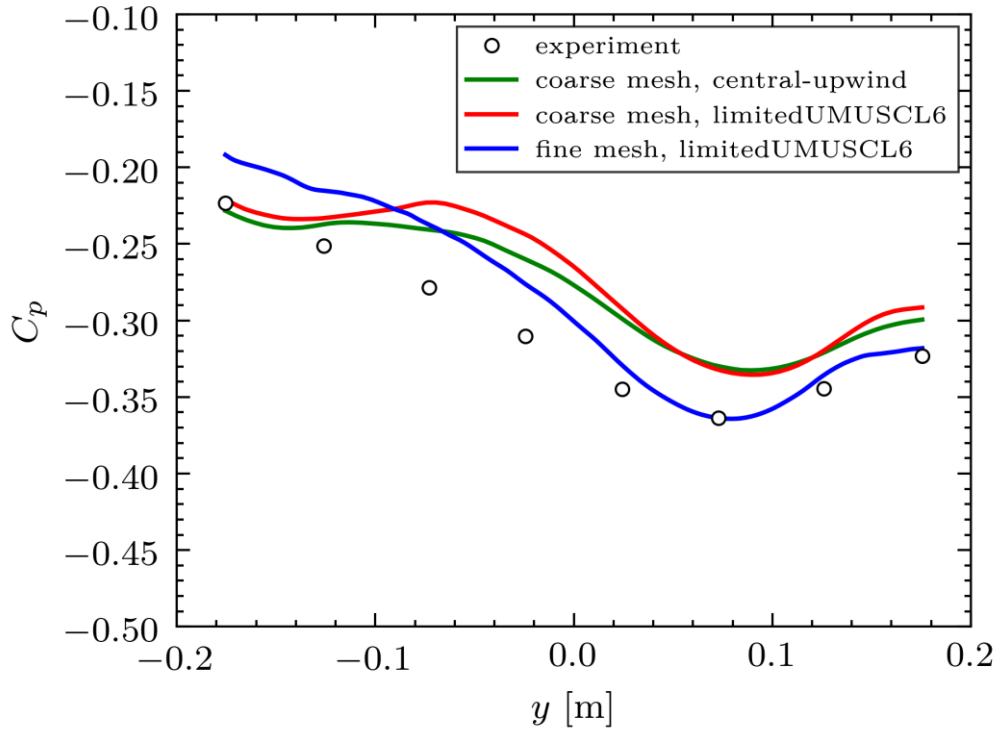


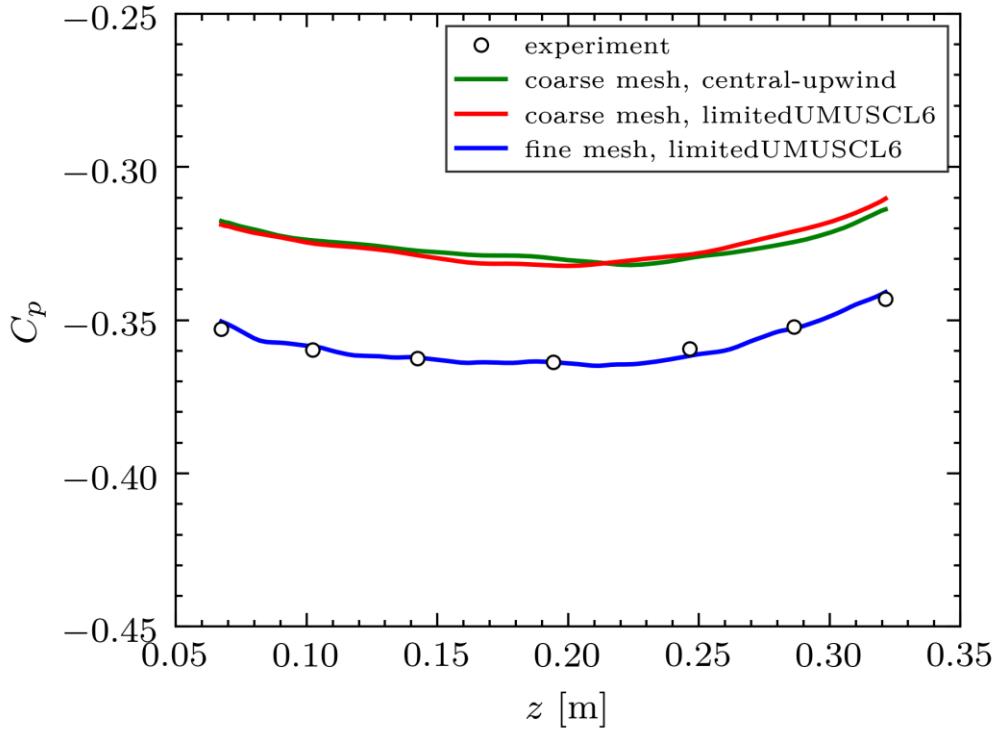
Windsor body – Group 1 centreline pressure

- The mean pressure coefficient is compared along the vehicle centreline.
- Blended 2nd order scheme and limitedUMUSCL6 give very similar results.
- The finer g2 result also overlaps closely with the g1 predictions.
- Centreline pressure shows limited sensitivity to the scheme and mesh resolution.



Windsor body – Group 4 base pressure





Horizontal cut at $z = 0.1944$ m and vertical cut at $y = 0.0729$ m.

Both schemes give similar distributions on g1. Greater sensitivity is found to wake resolution.

Windsor body – Aerodynamics coefficients

- limitedUMUSCL6 increases C_D by 2.3% on g1, bringing it closer to the experimental value.
- Pressure drag changes by only 1%, while the wall-shear contribution increases by approximately 10%.
- Refinement from g1 to g2 changes the total drag by less than 1%.
- Lift, side force and pitching moment show no consistent dependence on the scheme or mesh.

		central-upwind	limitedUMUSCL6	
Coefficient	**Experiment**	**g1**	**g1**	**g2**
C_D	0.3298	0.3017	0.3086	0.3107
$C_{D,p}$	–	0.2599	0.2625	0.2607
$C_{D,\tau}$	–	0.04185	0.04610	0.0500
C_L	−0.0382	−0.1739	−0.1429	−0.1552
C_S	0.1509	0.1552	0.1466	0.1679
C_{Pitch}	−0.0259	−0.0813	−0.0942	−0.0909

Conclusions

- UMUSCL6 extends the original UMUSCL reconstruction through successive differentiation, producing a sixth-order central operator for linear equations on uniform grids.
- limitedUMUSCL6 applies curvature-aware TVD clipping to the completed high-order face value.
- Smooth scalar transport confirms sixth-order convergence for UMUSCL6. Limiting reduces the formal order to second but retains substantially lower errors.
- In channel flow, limitedUMUSCL6 reduces the log-layer mismatch.
- For the Windsor body, the scheme improves the wall-shear contribution to drag, for a **30% increase in cost**.

limitedUMUSCL6 provides a robust, low-dissipation option for industrial WMLES, with potential benefits for aerodynamic prediction and the generation of higher-fidelity datasets for AI models.

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